ASSOCIAZIONE CULTURALE MATEMUPPER
Sede Legale: Via venosa, 10 – 20137 Milano (MI)
Sede Operativa: Via Bergamo, 7 – 20135 Milano (MI)
Codice Fiscale: 97785560158
Tel: +39 375 531 6594
e-mail: matemupper@matemupper.com
Web: matemupper.com

Esercizio 1

1. $$\int \frac{7}{6}\sqrt[6]{x}dx = \int \frac{7}{6}x^{\frac{1}{6}}dx = \frac{7}{6}\int x^{\frac{1}{6}}dx = \frac{7}{6}\left[\frac{1}{1+\frac{1}{6}}x^{1+\frac{1}{6}}\right] + C = \frac{7}{6}\frac{6}{7}x^{\frac{7}{6}} + C = \sqrt[6]{x^7} + C$$

2. $$\int e^{x+2}dx = \int e^2 e^x dx = e^2\int e^x dx = e^2 e^x + C = e^{x+2} + C$$

3. $$\int 4x^5 dx = 4\int x^5 dx = 4\frac{1}{1+5}x^{1+5} + C = \frac{2}{3}x^6 + C$$

4. $$\int \cos(x)dx = \sin(x) + C$$

Esercizio 2

1. $$\int [\cos(x) - \sin(x)]dx = \int \cos(x)dx - \int \sin(x)dx = \sin(x) + C - (-\cos(x) + D) = \sin(x) + \cos(x) + E$$

2. $$\int [x^3 + (x-1)^2]dx = \int x^3 dx + \int x^2 dx - 2\int x dx + \int 1dx = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + x + C$$

3. $$\int \frac{(x+1)(x+3)}{x}dx = \int \frac{x^2+4x+3}{x}dx = \int x dx + \int 4dx + \int \frac{3}{x}dx = \frac{1}{2}x^2 + 4x + 3\ln|x| + C$$

4. $$\int [3e^x + \tan(x) + 5]dx = 3\int e^x dx + \int \tan(x)dx + \int 5dx = 3e^x + \ln(|\cos(x)|) + 5x + C$$

Esercizio 3

1. $$\int \frac{4x}{2x^2+3}dx = \int \frac{[2x^2+3]'}{2x^2+3}dx = \ln|2x^2+3| + C$$

2. $$\int \frac{\cos(x)}{3\sin(x)+2}dx = \int \frac{3}{3}\frac{\cos(x)}{3\sin(x)+2}dx = \frac{1}{3}\int \frac{[3\sin(x)+2]'}{3\sin(x)+2}dx = \frac{1}{3}\ln|3\sin(x)+2| + C$$

3. $$\int \frac{x}{\sqrt{x^2+3}}dx = \int \frac{2}{2}\frac{x}{\sqrt{x^2+3}}dx = \frac{1}{2}\int \frac{[x^2+3]'}{(x^2+3)^{\frac{1}{2}}}dx = \frac{1}{2}\frac{1}{-\frac{1}{2}+1}(x^2+3)^{-\frac{1}{2}+1} + C = (x^2+3)^{\frac{1}{2}} + C$$

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4. $$\int \cos(3x)dx = \int \frac{3}{3}\cos(3x)dx = \frac{1}{3}\int 3\cos(3x)dx = \frac{1}{3}\sin(3x) + C$$

5. $$\int 4(4x+1)^2 dx = \frac{1}{3}(4x+1)^3 + C$$

6. $$\int \frac{\ln(x)}{x}dx = \int \ln x \left[\ln x\right]' dx = \frac{1}{2}\ln^2(x) + C$$

7. $$\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}dx = \int \sin(\sqrt{x}) \left[\sqrt{x}\right]' dx = \frac{1}{2}\sin^2(\sqrt{x}) + C$$

8. $$\int 4xe^{2x^2}dx = \int \left[2x^2\right]' e^{2x^2}dx = e^{2x^2} + C$$

Esercizio 4

1. $$\int \underset{f'}{x}\underset{g}{\ln(x)}dx = \frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C = \frac{x^2}{2}(\ln x - \frac{1}{2}) + C$$

2. $$\int \underset{g}{x}\underset{f'}{e^x}dx = xe^x - \int e^x dx = xe^x - e^x + C = e^x(x-1) + C$$

3. $$\int \underset{f'}{\ln(x)}\underset{g}{\ln(x)} = (x-1)\ln x \ln x - \int (x-1)\ln x \frac{1}{x}dx = (x-1)\ln^2(x) - \int \ln(x)dx - \int \frac{\ln(x)}{x}dx =$$
$$(x-1)\ln^2(x) - (x-1)\ln(x) - \frac{1}{2}\ln^2(x) + C = x(\ln^2(x) - 2\ln(x) + 2) + C$$

4. $$\int \underset{g}{x^2}\underset{f'}{e^x}dx = x^2e^x - \int 2xe^x = x^2e^x - 2(e^x(x-1)) + C = e^x(x^2 - 2x - 2) + C$$

Quest'ultimo integrale è identico al precedente, come mai? (*hint*: poniamo $t = \ln(x)$, cosa succede?)